BC Calc WK8 Tuesday

Name ____________________

11. Suppose that function $f(x)$ is approximated near $x=0$ by a sixth-degree Taylor polynomial $P_6(x)=3x-4x^3+5x^6$. Give the value of each of the following:

(a) $f(0)$ (b) $f'(0)$ (c) $f'''(0)$ (d) $f^{(5)}(0)$ (e) $f^{(6)}(0)$

12. (Calculator Permitted) Suppose that g is a function which has continuous derivatives, and that

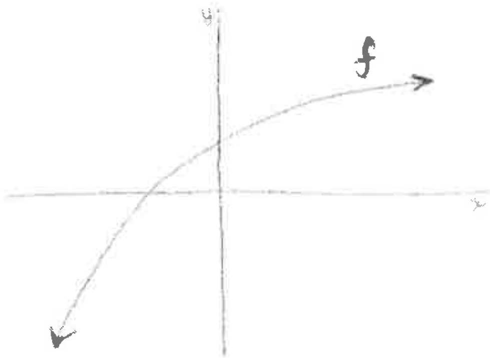
$$g(5)=3,\ g'(5)=-2,\ g''(5)=1,\ g'''(5)=-3$$

(a) What is the Taylor polynomial of degree 2 for g near 5? What is the Taylor polynomial of degree 3 near 5?

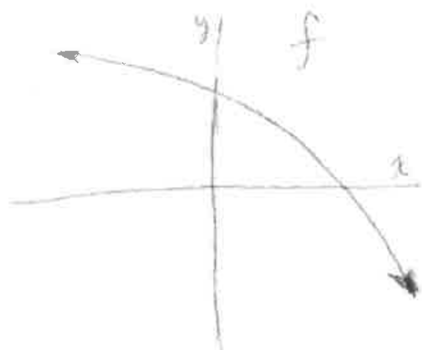
(b) Use the two polynomials that you found in part (a) to approximate $g(4.9)$.

For problems 13-16, suppose that $P_2(x) = a + bx + cx^2$ is the second degree Taylor polynomial for the function *f* about $x = 0$. What can you say about the signs of *a*, *b*, and *c*, if *f* has the graphs given below?

13.



14.



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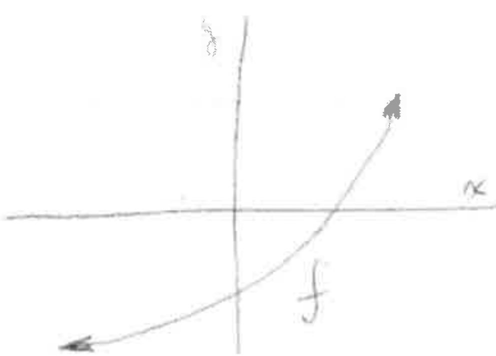
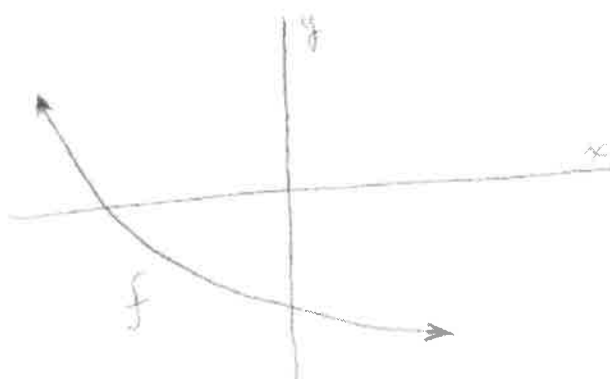


16.



17. Show how you can use the Taylor approximation $\sin x \approx x - \frac{x^3}{3!}$ for x near 0 to find $\lim_{x \to 0} \frac{\sin x}{x}$.

18. Use the fourth-degree Taylor approximation of $\cos x \approx 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$ for x near 0 to find $\lim_{x \to 0} \frac{1 - \cos x}{x}$.

19. Estimate the integral $\int_0^1 \frac{\sin t}{t} dt$ using a Taylor polynomial for $\sin t$ about $t = 0$ of degree 5.

Multiple Choice

20. If $f(0)=0$, $f'(0)=1$, $f''(0)=0$, and $f'''(0)=2$, then which of the following is the third-order Taylor polynomial generated by $f(x)$ at $x=0$?

(A) $2x^3+x$ (B) $\frac{1}{3}x^3+\frac{1}{2}x$ (C) $\frac{2}{3}x^3+x$ (D) $2x^3-x$ (E) $\frac{1}{3}x^3+x$

21. Which of the following is the coefficient of x^4 in the Maclaurin polynomial generated by $\cos(3x)$?

(A) $\frac{27}{8}$ (B) 9 (C) $\frac{1}{24}$ (D) 0 (E) $-\frac{27}{8}$

22. Which of the following is the Taylor polynomial generated by $f(x)=\cos x$ at $x=\frac{\pi}{2}$?

(A) $\left(x-\frac{\pi}{2}\right)-\frac{\left(x-\frac{\pi}{2}\right)^3}{3!}+\frac{\left(x-\frac{\pi}{2}\right)^4}{4!}$ (B) $1+\frac{\left(x-\frac{\pi}{2}\right)^2}{2!}+\frac{\left(x-\frac{\pi}{2}\right)^4}{4!}$ (C) $1-\frac{\left(x-\frac{\pi}{2}\right)^2}{2!}+\frac{\left(x-\frac{\pi}{2}\right)^4}{4!}$

(D) $1-\left(x-\frac{\pi}{2}\right)^2+\left(x-\frac{\pi}{2}\right)^4$ (E) $-\left(x-\frac{\pi}{2}\right)+\frac{\left(x-\frac{\pi}{2}\right)^3}{6}$

23. (Calculator Permitted) Which of the following gives the Maclaurin polynomial of order 5 approximation to $\sin(1.5)$?

(A) 0.965 (B) 0.985 (C) 0.997 (D) 1.001 (E) 1.005

24. Which of the following is the quadratic approximation for $f(x)=e^{-x}$ at $x=0$?

(A) $1-x+\frac{1}{2}x^2$ (B) $1-x-\frac{1}{2}x^2$ (C) $1+x+\frac{1}{2}x^2$ (D) $1+x$ (E) $1-x$